Quantumn Information Theory

$$|xy\rangle \xrightarrow{E} |\beta_{xy}\rangle \xrightarrow{E^{-1}} |xy\rangle$$

∘ $\begin{cases}\text{Bell} \\ \text{EPR}\end{cases}$ States (測量一個 qubit, 瞬間決定另一個), $|\beta_{xy}\rangle = \frac{1}{\sqrt{2}}\left(|0y\rangle + (-1)^{x}|1\bar{y}\rangle\right)$

$$\begin{cases}
|00\rangle \longrightarrow |\beta_{00}\rangle = \dfrac{|00\rangle + |11\rangle}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\1\end{bmatrix} \\
|01\rangle \longrightarrow |\beta_{01}\rangle = \dfrac{|01\rangle + |10\rangle}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\1\\0\end{bmatrix} \\
|10\rangle \longrightarrow |\beta_{10}\rangle = \dfrac{|00\rangle - |11\rangle}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\-1\end{bmatrix} \\
|11\rangle \longrightarrow |\beta_{11}\rangle = \dfrac{|01\rangle - |10\rangle}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\-1\\0\end{bmatrix}
\end{cases}$$

$$\begin{cases}
|00\rangle = \frac{1}{\sqrt{2}}\left(|\beta_{00}\rangle + |\beta_{10}\rangle\right) \\
|01\rangle = \frac{1}{\sqrt{2}}\left(|\beta_{01}\rangle + |\beta_{11}\rangle\right) \\
|10\rangle = \frac{1}{\sqrt{2}}\left(|\beta_{01}\rangle - |\beta_{11}\rangle\right) \\
|11\rangle = \frac{1}{\sqrt{2}}\left(|\beta_{00}\rangle - |\beta_{10}\rangle\right)
\end{cases}$$

(1) $|\beta_{xy}\rangle$ entangled states

$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) = \frac{1}{\sqrt{2}}\left(a|0\rangle + b|1\rangle\right)\left(c|0\rangle + d|1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle\right)$$

$$ad = 0 \Rightarrow \begin{cases} a = 0, & ac = 0 \quad (\times) \\ d = 0, & bd = 0 \quad (\times) \end{cases}$$

(2) $\{|\beta_{00}\rangle, |\beta_{01}\rangle, |\beta_{10}\rangle, |\beta_{11}\rangle\}$ orthonormal basis.

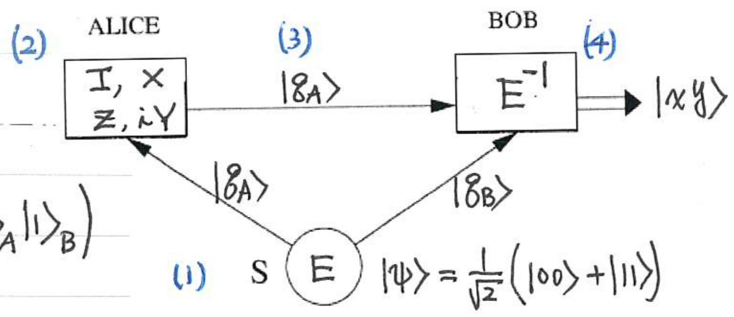
Pauli operators

名稱	作用	$a\|0\rangle + b\|1\rangle = \begin{bmatrix} a \\ b \end{bmatrix} \to ?$	
$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{cases} \|0\rangle \to \|0\rangle \\ \|1\rangle \to \|1\rangle \end{cases}$	$a\|0\rangle + b\|1\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$	
$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{cases} \|0\rangle \to \|1\rangle \\ \|1\rangle \to \|0\rangle \end{cases}$	$a\|1\rangle + b\|0\rangle = \begin{bmatrix} b \\ a \end{bmatrix}$	
$iY = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$	$\begin{cases} \|0\rangle \to -\|1\rangle \\ \|1\rangle \to \|0\rangle \end{cases}$	$-a\|1\rangle + b\|0\rangle = \begin{bmatrix} b \\ -a \end{bmatrix}$,	$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{cases} \|0\rangle \to \|0\rangle \\ \|1\rangle \to -\|1\rangle \end{cases}$	$a\|0\rangle - b\|1\rangle = \begin{bmatrix} a \\ -b \end{bmatrix}$	$ZX = iY$

Superdense coding

transmit 2 classical bits (xy), by sending a qubit ($|\gamma_A\rangle$).

procedure:



(1) Source 產生 $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

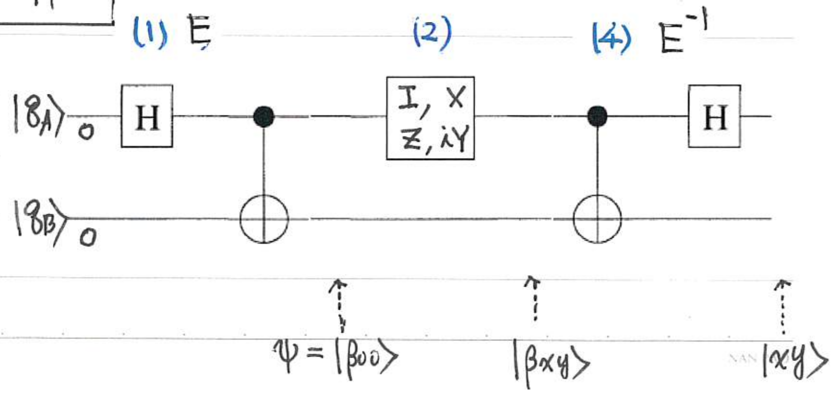
$$= \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$$

(2) To transmit xy, A applies to $|\gamma_A\rangle$

xy	OP	$\frac{1}{\sqrt{2}}(\|00\rangle + \|11\rangle) \to \|\beta_{xy}\rangle$
00	I	$\frac{1}{\sqrt{2}}(\|00\rangle + \|11\rangle) = \|\beta_{00}\rangle$
01	X	$\frac{1}{\sqrt{2}}(\|10\rangle + \|01\rangle) = \|\beta_{01}\rangle$
10	Z	$\frac{1}{\sqrt{2}}(\|00\rangle - \|11\rangle) = \|\beta_{10}\rangle$
11	iY	$\frac{1}{\sqrt{2}}(-\|10\rangle + \|01\rangle) = \|\beta_{11}\rangle$

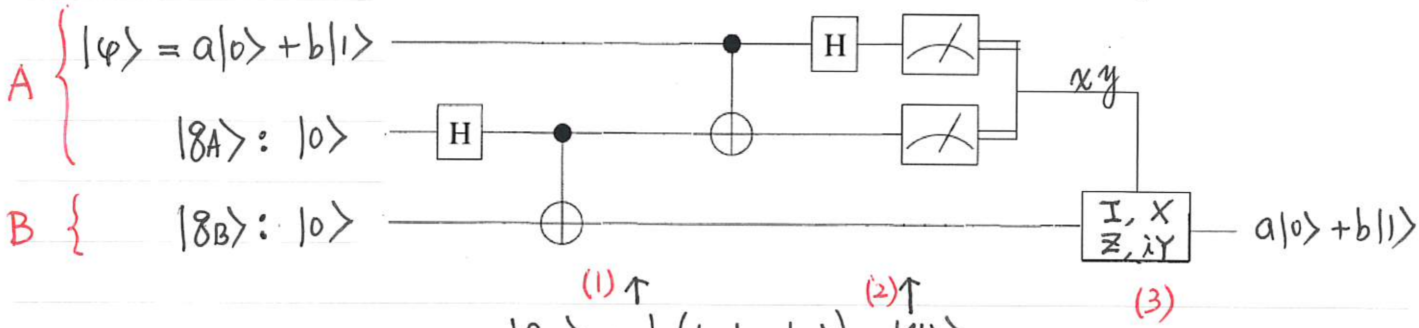
(3) A send $|\gamma_A\rangle$ to B

(4) B: $|\beta_{xy}\rangle \xrightarrow{E^{-1}} |xy\rangle$



Quantumn Teleportation

Transmit 1 qubit $|\psi\rangle = a|0\rangle + b|1\rangle$, by sending 2 classical bits



$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) \qquad |\psi\rangle$$
$$= \frac{1}{\sqrt{2}}\left(|0\rangle_A |0\rangle_1 + |1\rangle_A |1\rangle_B\right)$$

(1)

$$|\varphi\rangle|\beta_{00}\rangle = \left(a|0\rangle + b|1\rangle\right)\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$
$$= \frac{1}{\sqrt{2}}\left(a|000\rangle + a|011\rangle + b|100\rangle + b|111\rangle\right)$$
$$= \frac{1}{\sqrt{2}}\left(a|00\rangle_A |0\rangle_B + a|01\rangle_A |1\rangle_B + b|10\rangle_A |0\rangle_B + b|11\rangle_A |1\rangle_B\right)$$
$$= \frac{1}{\sqrt{2}}\Big[\frac{a}{\sqrt{2}}\left(|\beta_{00} + \beta_{10}\right)|0\rangle + \frac{a}{\sqrt{2}}\left(|\beta_{01}\rangle + |\beta_{11}\rangle\right)|1\rangle$$
$$+ \frac{b}{\sqrt{2}}\left(|\beta_{01}\rangle - |\beta_{11}\rangle\right)|0\rangle + \frac{b}{\sqrt{2}}\left(|\beta_{00}\rangle - |\beta_{10}\rangle\right)|1\rangle\Big]$$
$$= \frac{1}{2}\Big[|\beta_{00}\rangle\left(a|0\rangle + b|1\rangle\right) + |\beta_{01}\rangle\left(a|1\rangle + b|0\rangle\right) + |\beta_{10}\rangle\left(a|0\rangle - b|1\rangle\right)$$
$$+ |\beta_{11}\rangle\left(a|1\rangle - b|0\rangle\right)\Big]$$

(2)

$$\xrightarrow{E^{-1}} |\psi\rangle = \frac{1}{2}\left\{\begin{array}{lll} |00\rangle\left(a|0\rangle + b|1\rangle\right) : \begin{bmatrix} a \\ b \end{bmatrix} & \xrightarrow{I} & \\ + |01\rangle\left(a|1\rangle + b|0\rangle\right) : \begin{bmatrix} b \\ a \end{bmatrix} & \xrightarrow{X} & a|0\rangle + b|1\rangle = \begin{bmatrix} a \\ b \end{bmatrix} \\ + |10\rangle\left(a|0\rangle - b|1\rangle\right) : \begin{bmatrix} a \\ -b \end{bmatrix} & \xrightarrow{Z} & \\ + |11\rangle\left(a|1\rangle - b|0\rangle\right) : \begin{bmatrix} -b \\ a \end{bmatrix} & \xrightarrow{iY} & \end{array}\right.$$

$$A : \underbrace{xy}\ \underbrace{\longrightarrow}\ B$$